Networks of perfect tensors and symplectic geometry over finite fields

(in the context of p-adic AdS/CFT)

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Arithmetic ideas seem to come up in physics in surprising / natural ways!

- AdS/CFT
- (perfect) tensor network models
- a bit about p-adic AdS/CFT
- what does "perfect" mean physically?
- finite fields

⑦ Generalities on AdS/CFT

$\mathcal{N} = 4$ Super Yang Mills $\longleftrightarrow$ type IIB SUGRA on $AdS_5 \times S^5$

w/ $G = U(N)$ — w/ N units of 5-form flux.

(Witten)

boundary		bulk
S^n		H^{n+1}, $\partial H^{n+1} = S^n$
$\mathrm{Conf}(n)$	$\cong$	$\mathrm{Iso}(H^{n+1})$
operators		fields ⊛

[⊛ Koszul duality; see Costello; Li, Paquette ...]

(conformal) field theory	gravity
strong coupling	weak coupling

One main subject in the development of this dictionary has been

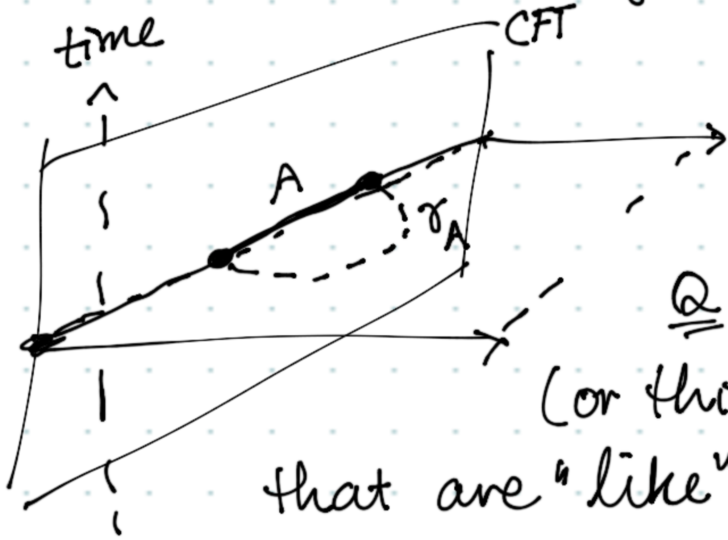
<u>entanglement entropy</u>.

Bekenstein/Hawking:

$$S_{BH} \sim \frac{\text{Area}(\Sigma)}{4}$$

$\Rightarrow$ Degrees of freedom in QG scale weirdly!

Generalization: Ryu-Takayanagi



Entanglement entropy

$$S_A \sim \text{Area}(\gamma_A)$$

Q: How can we build (or think about) states that are "like" the CFT ground state?

One answer: tensor networks. (Many versions..)

I'll just discuss one example (Harlow, Preskill, Pastawski, Yoshida)

Fix a f.d. Hilbert space V

Choose a perfect tensor

$$\psi \in V^{\otimes d}.$$

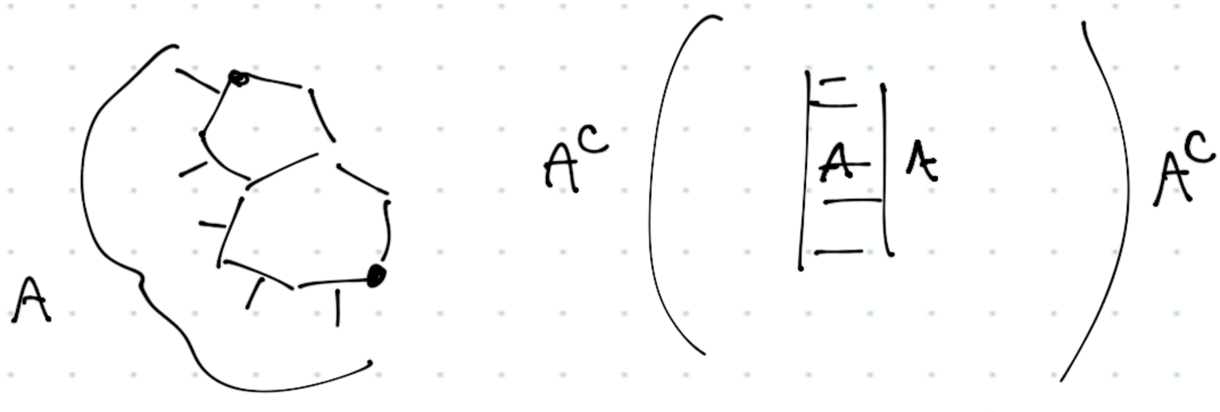
ψ is perfect when, as a map

$$\psi : V^{\otimes k} \to V^{\otimes (d-k)} \quad (k \leq d/2)$$

it is always an isometric embedding.

$\Rightarrow$ Pick any "tiling" of any geometry whatsoever, and decorate each tile with ψ. Glue (contract indices) along common faces.

What's nice about this?
Minimal surface property is obvious!!



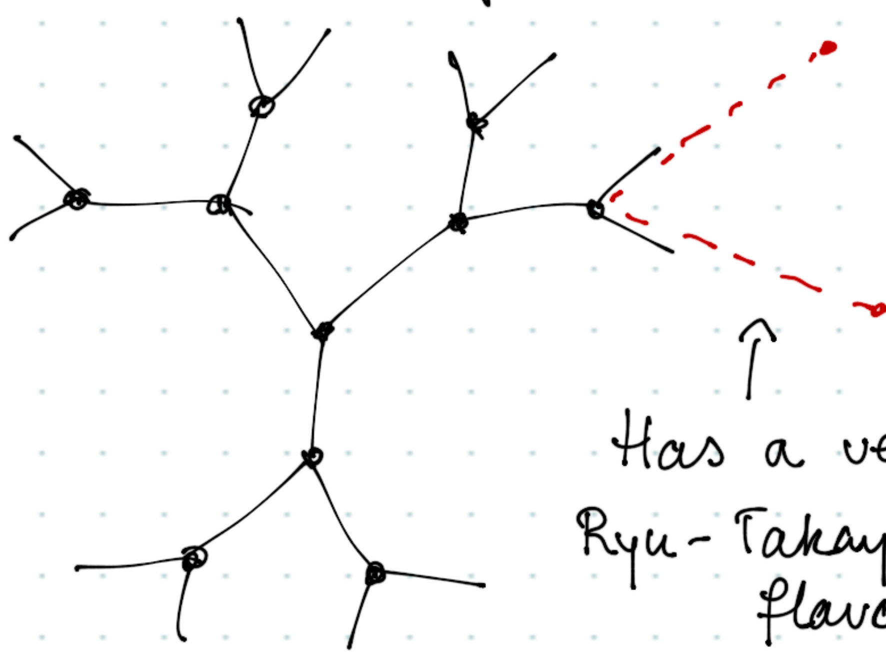
$$\Rightarrow S \sim \dim(\text{smallest } \mathcal{H}) \sim \text{min surface.} \qquad \square$$

It was the excitement about such models that led me to start thinking about p-adic AdS/CFT ...

There's a natural discretization of bulk/boundary geometry via p-adic symmetric spaces.

boundary		bulk
$\mathbb{P}^1(\mathbb{Q}_p)$	$=\partial$	$\underline{\underline{T_p}}$ $\left(T_p = PGL(2,\mathbb{Q}_p) \big/ PGL(2,\mathbb{Z}_p)\right)$

$$\text{"}Conf\text{"}(\mathbb{P}^1) = \text{"}Iso\text{"}(T_p) = PGL(2,\mathbb{Q}_p).$$



Has a very Ryu-Takayanagi flavor...

Problem: a perfect tensor network wants to be "dual" to T_p, rather than live on T_p itself.

What can we do to think about "perfectness" in a more "physical" way?

<u>Idea</u>: How should I think of V (and operators on V)?

If V is the quantization of a phase space, it must be compact.

But I'd rather think about "flat" phase spaces...

$$V = \mathcal{O}(\mathbb{F}_q)$$

is the quantization of $(\mathbb{F}_q^2, \omega)$!

Operators are $\mathrm{Heis}(\mathbb{F}_q^2, \omega) \curvearrowright V$

$$\underline{0 \to \mathbb{F}_q \to \mathrm{Heis}(\mathbb{F}_q^2, \omega) \to \mathbb{F}_q^2} \to 0$$

A dream about quantization:
it "might be" a functor from

(sympl. manifolds, Lag. correspondences) $\to$ Hilb.
(Wehrheim, Woodward, ...)

Very false!

But:

- there is a linear Weinstein category

(sympl. vector spaces, linear Lag. corresp.)

- Over finite fields, there is even a "canonical quantization" functor! (Gurevich, Hadani)

A perfect tensor is a Lagrangian subspace of $(\mathbb{F}_q^{2d}, \omega)$ that's in general position.

$$\mathcal{H}((\mathbb{F}_q^2)^{\oplus d}) = \mathcal{H}(\mathbb{F}_q^2)^{\otimes d}$$
$$\cong$$
$$V.$$